which are all +ve.

So, $H(X)$ is positive Definite.

i.e. $f(X)$ is convex.

and Hence $f(X)$ is min^m at $(5, 11, 4)$

(Page 830 and 831).

Q. Solve

$$\max. \ Z = x_1^2 + 4x_1x_2 + x_2^2$$

s.t.

$$x_1^2 + x_2^2 = 1$$

and $x_1, x_2 \geq 0$.

Ans. $\rightarrow$ $x_1 = \frac{1}{\sqrt{2}}$, $x_2 = \frac{1}{\sqrt{2}}$

$\max. Z = 3$.

Q). $\max. Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

s.t.

$$x_1 + 2x_2 = 2$$

and $x_1, x_2 \geq 0$

Q. Solve

$$\min\ z = x_1^2 + x_2^2 + x_3^2$$

s.t.

$$x_1 + x_2 + 3x_3 = 2$$

$$5x_1 + 2x_2 + x_3 = 5$$

$$x_1, x_2, x_3 \geq 0$$

by Applying Lagrangian multipliers.

Solⁿ:

Here, $f(x) = x_1^2 + x_2^2 + x_3^2$

$g_1(x) = x_1 + x_2 + 3x_3 = 2$

$g_2(x) = 5x_1 + 2x_2 + x_3 = 5$

$\Rightarrow h_1(x) = x_1 + x_2 + 3x_3 - 2 = 0$

$h_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0.$

The Lagrangian function is

$$L(x, \lambda) = f(x) - \lambda_1 h_1(x) - \lambda_2 h_2(x).$$

$$= x_1^2 + x_2^2 + x_3^2 - \lambda_1 [x_1 + x_2 + 3x_3 - 2]$$

$$- \lambda_2 [5x_1 + 2x_2 + x_3 - 5]$$

The necessary condⁿ for $f(x)$ to be max^{m} or min^{m} are

$$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 2x_1 - \lambda_1 - 5\lambda_2 = 0 \quad \text{---} \; (1)$$

$$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 2x_2 - \lambda_1 - 2\lambda_2 = 0 \quad \text{---} \; (2)$$

$$\frac{\partial L}{\partial x_3} = 0 \Rightarrow 2x_3 - 3\lambda_1 - \lambda_2 = 0 \quad \text{---} \; (3)$$

$$\frac{\partial L}{\partial \lambda_1} = 0 \Rightarrow x_1 + x_2 + 3x_3 - 2 = 0 \quad \text{---} \; (4)$$

$$\frac{\partial L}{\partial \lambda_2} = 0 \Rightarrow 5x_1 + 2x_2 + x_3 - 5 = 0 \quad \text{---} \; (5)$$

from (1), $x_1 = \frac{1}{2}[\lambda_1 + 5\lambda_2]$

from (2), $x_2 = \frac{1}{2}[\lambda_1 + 2\lambda_2]$

from (3), $x_3 = \frac{1}{2}[3\lambda_1 + \lambda_2]$

putting the values of x_1, x_2 and x_3 in eqⁿ (4)

$$\frac{1}{2}[\lambda_1 + 5\lambda_2 + \lambda_1 + 2\lambda_2 + 9\lambda_1 + 3\lambda_2] - 2 = 0$$

$$\Rightarrow 11\lambda_1 + 10\lambda_2 = 4 \quad \text{---} \; (6)$$

putting the values of x_1, x_2 and x_3 in eq^n 5

$$\frac{1}{2}\left[5\lambda_1 + 25\lambda_2 + 2\lambda_1 + 4\lambda_2 + 3\lambda_1 + \lambda_2\right] - 5 = 0$$

$$\Rightarrow 10\lambda_1 + 30\lambda_2 = 10$$

i.e. $\lambda_1 + 3\lambda_2 = 1$ —— 7

solving 6 and 7.

$$11(1-3\lambda_2) + 10\lambda_2 = 4$$

or, $11 - 33\lambda_2 + 10\lambda_2 = 4$

or, $-23\lambda_2 = -7$

$$\boxed{\lambda_2 = \frac{7}{23}}$$

$$\therefore \lambda_1 = 1 - 3\lambda_2$$

$$= 1 - 3 \times \frac{7}{23}$$

$$\boxed{\lambda_1 = \frac{2}{23}}$$

$$\therefore x_1 = \frac{1}{2}\left[\frac{2}{23} + 5 \times \frac{7}{23}\right] = \frac{37}{46}$$

$$x_2 = \frac{1}{2}\left[\frac{2}{23} + 2 \times \frac{7}{23}\right] = \frac{16}{46} = \frac{8}{23}$$

$$x_3 = \frac{1}{2}\left[3 \times \frac{2}{23} + \frac{7}{23}\right] = \frac{13}{46}$$

$\therefore$ The stationary point is $\left(\frac{37}{46}, \frac{8}{23}, \frac{13}{46}, \frac{2}{23}, \frac{7}{23}\right)$

Now,
max or min. of $f(x)$ can be checked as follows:

Hessian matrix of $f(x)$ at $\left(\frac{37}{46}, \frac{8}{23}, \frac{13}{46}\right)$ is

$$H(X) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$\therefore$ Principal minor of $H(X)$ of order one $= |2| = 2 > 0$

of order two $= \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$

and of order three $= \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$

$= 8 > 0$

which are all +ve.

So, $H(X)$ is +ve definite

i.e. $f(X)$ is convex.

Hence $f(X)$ is min^m at $\left(\frac{37}{46}, \frac{8}{23}, \frac{13}{46}\right)$

and $\min. f(X) = \min Z$

$= 0.857.$

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